MLIR ODM
Polynomial Dialect

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Agenda

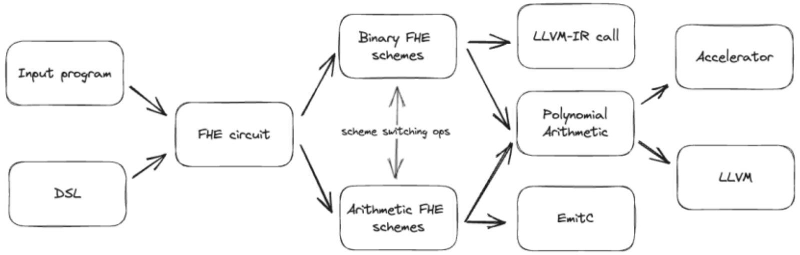
- Motivation
- Types & Attributes
- Ops & Lowerings
- Roadmap

Motivation

tl;dr: hardware acceleration for cryptography

Cryptographic applications

- "FHE": Fully Homomorphic Encryption (compute on encrypted data)
  - HEIR
  - HECO
  - HECATE
  - Concrete
  - HEaaN.MLIR



- NIST-standard Post-Quantum Cryptography (PQC)
  - Kyber (key encapsulation)
  - Dilithium (digital signature)

Bottleneck: modular polynomial arithmetic

$$\mathbb{Z}_q[x]/(\,F(x)\,)$$

Polynomial modular arithmetic

$Example$: $q = 8, \qquad F(x) = x^3 + 1$

$$(1 + 5x + x^2)(3 + x - x^2)$$

Normal product: $$3 + 16x + 7x^2 - 4x^3 - x^4$$

Mod $x^3 + 1$: $$7 + 17x + 7x^2$$ "set" $x^3 + 1 = 0$ and reduce

Mod 8 coefficients : $$7 + x + 7x^2$$

Choice of polynomial & coefficient mod is security + performance critical

$FHE\ Example$: $q \approx 952809000096560291, \qquad F(x) = x^{65536} + 1$

Why put crypto in the compiler?

- Need hardware acceleration for FHE and PQC!
  - GPU/TPU
  - FPGA
  - Optical accelerators
- HW supports poly ops as first-class operations
  - DARPA DPRIVE

Input + computation dialect
No new MLIR infrastructure
Lowers to existing dialects

Prototype implementation at
github.com/google/heir

Types and Attributes

tl;dr: a new polynomial type

Polynomial type

```
!polynomial.polynomial<#ring>
```

A polynomial is an element of a ring*

```
#ring = #polynomial.ring<
        ctype=i32,
        cmod=4294967291,
        ideal=#p
      >
```

Coefficient (base) type

Coefficient modulus

Polynomial modulus

```
#p = #polynomial.polynomial<1 + x**1024>
```

Modulus must be statically known
Custom attribute: parser, storage

*"ring" is the math name for this

Why a new type?

- are polynomials just “tensors with metadata“?
- A polynomial can be stored in many ways
  - Many nonzero coefficients -> dense tensor
  - Large degree and many zero coefficients -> sparse tensor
  - Not necessarily always a list of coefficients (DFT/NTT “evaluation form”)
- Polynomial seems like the right abstraction for passes

Why specify ring on the type?

- "Better for ops to specify semantics“
- Once you agree we need a new type...
- Type conversion needs to pick `tensor<dim x ty>`
- Alternatives to make type conversion work seem overkill

Why not hard-code choices?

- Why not specialize to $(x^N + 1)$
  - Future proof for new innovations in PQC/FHE crypto
  - Support non-PQC crypto (e.g. secret sharing)
  - Support non-crypto scientific computation
  - Potential use within MLIR in polyhedral analysis
- Post-Quantum Crypto, Private Information Retrieval, Secure Multi-Party Computation
  - Similar crypto-friendly polynomial math
  - Often much smaller security parameters than FHE -> different tradeoffs

Ops & Passes

tl;dr: optimized lowerings for common rings

Obvious `polynomial` ops

- `constant <1 + x**1024>`
- `add`, `sub`, `mul`, `div_rem*`
- `to_tensor`, `from_tensor`
- `mul_scalar`
- `leading_term` (degree + leading coefficient)

Less obvious ops

- `monomial` construct a single-term polynomial from data
- `monomial_mul` multiply a polynomial by a monomial (optimized lowerings)
- `dft/idft` compute a forward/reverse complex Fourier transform
- `ntt/intt` compute a forward/reverse integer number theoretic transform (integer-only DFT analogue)

Enables O(n log(n)) multiplication of $f * g$ as $iNTT\big(NTT(f) \cdot NTT(g)\big)$ with $\cdot$ elementwise!

While this requires a compatible ring, virtually all crypto uses such rings

Ops we probably don't need

- `tensor_mul` use linalg.generic with poly ops inside?

Lowering mul

- Generic lowering supporting all* parameters: Naive polymul + modular reduction
  - Naive polymul computed via...
    - Cyclic convolution (linalg.generic)
    - DFT/NTT + pointwise mul + IDFT/INTT
    - Karatsuba, etc.
  - Modular reduction via textbook poly long division (scf.while)
- $\mathbb{Z}_q[x]/(x^N + 1)$ - machine word-sized coefficients
  - no manual mod reduction step
  - (Nega)cyclic convolution
  - DFT + entry-wise mul + IDFT
  - Tensor mul via Toeplitz matrix trick (TPU)
- $\mathbb{Z}_p[x]/(x^N + 1)$ - prime coefficients (< 64-bits)
  - NTT + entry-wise mul + INTT

Lowering dft/ntt

- Goal: keep polynomial in coefficient or evaluation form for as long as possible
  - Canonicalize `[dft, op1, idft, dft, op2, idft]` to `[dft, op1, op2, idft]`
- Lowering via
  - Cooley-Tukey
  - [Stockham + AVX]()
  - Dedicated accelerator support

Formal verification!

- Cambridge group (Tobias Grosser) looking into Lean to formalize MLIR dialects & passes
- Polynomial dialect is one of their case studies
- We hope this will allow us to formally verify correctness of polynomial passes and lowerings

Roadmap

tl;dr: lower to LLVM, then accelerators

Milestones

Lowering polynomial to LLVM via standard dialects

Generic Ring	$\mathbb{Z}_p[x]/(x^N + 1)$

Nontrivial example implementations as end-to-end tests

Polynomial interpolation	Simplified Dilithium scheme

Focus on HW acceleration

GitHub repo
github.com/google/heir

HEIR meetings
google.github.io/heir/community/

RFC on Discourse
shorturl.at/fNO18

Questions